# Multifactor RSM Tutorial

## Combining categorical with numeric factors

Procter and Gamble (P&G) engineers working on a new sealing process became concerned about how maximum peel strength would be affected by changing suppliers of packaging material (William A Brenneman. and William R. Myers, "Robust Parameter Design with Categorical Noise Variables," *Journal of Quality Technology* 35, no. 4 (October 2003): 335-341). They set up an RSM design to vary several key factors on the sealing machine, including the supplier:

A. Temperature, 193 to 230 degrees C.

B. Pressure, 2.2 to 3.2 Bar.

C. Speed, 32 to 50 cpm (cycles per minute).

D. Supplier: S1, S2, S3.

Due to limitations of time and other resources, a maximum of 37 runs can be performed. Therefore, simply conducting a standard central composite design (CCD) or Box-Behnken design (BBD) for each of the three suppliers will not do – these design choices produce far too many runs (60 and 51, respectively). Instead, P&G engineers use a D-optimal design.

The data below came from a simulation loosely based on the predictive model reported in the cited article. (For tutorial purposes, some liberties are taken to make the procedural outcome more challenging.) Assume the maximum peel strength ideally hits a target of 4.5 pound-force (lbf). However, it must exceed 3 lbf to prevent leaking and not exceed 6 lbf because the package becomes too difficult to open.

This tutorial explains how a design is constructed via D-optimal criteria with added points for lack of fit testing and pure-error estimation. Notice we use nearly all budgeted runs. Also, aided by a feature in the software that forces balance, we make sure each supplier receives an equal number of design runs.

With guidance from the tutorial you analyze responses and then find desirable solutions meeting the following objectives – listed according to relative acceptability for all concerned:

1. Can the process be adjusted to attain a maximum peel strength target of 4.5 lbf for any one supplier?

2. The purchasing agent intends to break down suppliers as follows: 50% to S1, 25% to S2, and 25% to S3. Can you find a process setup that works for all suppliers? If not, a two-supplier option might be satisfactory, provided S1 is one of them. In other words, assuming the process is not robust for all three suppliers, can the process be set up in a manner that either S1-S2 or S1-S3 meets specifications?

3. If it is not possible to achieve a common process for even two of the three suppliers, perhaps the upper limit of 6 can be raised on the maximum peel strength specification. (Assume customers are willing to use scissors or,

better yet, package designers notch the plastic for easier opening.) Does this open a window of operability for multiple suppliers at one set of process conditions?

Here is the simulation data loosely based on the above cited article:

| Std | A: Temp (deg C) | B: Pressure (Bar) | C: Speed (cpm) | D: Supplier | Peel Strength (lbf) |
|---|---|---|---|---|---|
| 1 | 193 | 2.2 | 32 | S1 | 4.6 |
| 2 | 230 | 2.2 | 32 | S1 | 10.0 |
| 3 | 193 | 3.2 | 32 | S1 | 7.2 |
| 4 | 193 | 3.2 | 32 | S1 | 6.6 |
| 5 | 230 | 3.2 | 32 | S1 | 5.5 |
| 6 | 202.25 | 2.7 | 41 | S1 | 8.2 |
| 7 | 230 | 2.7 | 41 | S1 | 8.5 |
| 8 | 193 | 2.2 | 50 | S1 | 6.7 |
| 9 | 230 | 2.2 | 50 | S1 | 11.0 |
| 10 | 230 | 2.2 | 50 | S1 | 11.0 |
| 11 | 230 | 3.2 | 50 | S1 | 7.8 |
| 12 | 230 | 3.2 | 50 | S1 | 7.1 |
| 13 | 230 | 2.2 | 32 | S2 | 5.1 |
| 14 | 230 | 2.2 | 32 | S2 | 6.7 |
| 15 | 211.5 | 2.7 | 32 | S2 | 2.9 |
| 16 | 230 | 3.2 | 32 | S2 | 1.7 |
| 17 | 211.5 | 2.2 | 41 | S2 | 6.3 |
| 18 | 193 | 2.7 | 41 | S2 | 2.0 |
| 19 | 220.75 | 2.95 | 41 | S2 | 3.7 |
| 20 | 193 | 2.2 | 50 | S2 | 2.4 |
| 21 | 230 | 2.7 | 50 | S2 | 4.0 |
| 22 | 230 | 2.7 | 50 | S2 | 4.1 |
| 23 | 193 | 3.2 | 50 | S2 | 4.8 |
| 24 | 193 | 3.2 | 50 | S2 | 5.1 |
| 25 | 193 | 2.2 | 32 | S3 | 6.7 |
| 26 | 193 | 2.2 | 32 | S3 | 6.7 |
| 27 | 230 | 3.2 | 32 | S3 | 8.0 |
| 28 | 230 | 3.2 | 32 | S3 | 7.1 |
| 29 | 220.75 | 2.45 | 36.5 | S3 | 9.2 |
| 30 | 202.25 | 2.7 | 36.5 | S3 | 8.1 |
| 31 | 211.5 | 3.2 | 41 | S3 | 7.2 |
| 32 | 230 | 2.2 | 50 | S3 | 9.5 |
| 33 | 230 | 2.2 | 50 | S3 | 9.8 |
| 34 | 211.5 | 2.7 | 50 | S3 | 6.2 |
| 35 | 193 | 3.2 | 50 | S3 | 5.9 |
| 36 | 230 | 3.2 | 50 | S3 | 4.8 |

*Data based on package sealing experiment*

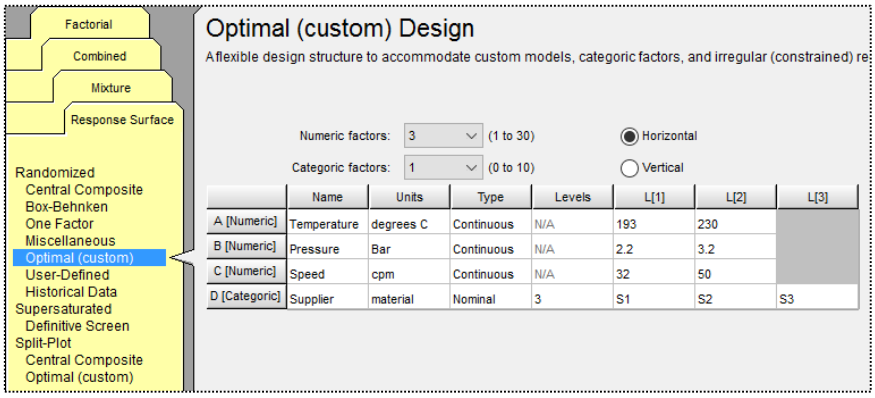
# Design the Experiment

Start Design-Expert® software and initiate your design process by clicking the blank-sheet icon 🗋 on the left of the toolbar, or select File, New Design, or simply click the **New Design** button on our opening screen.




*New easy-start design options*

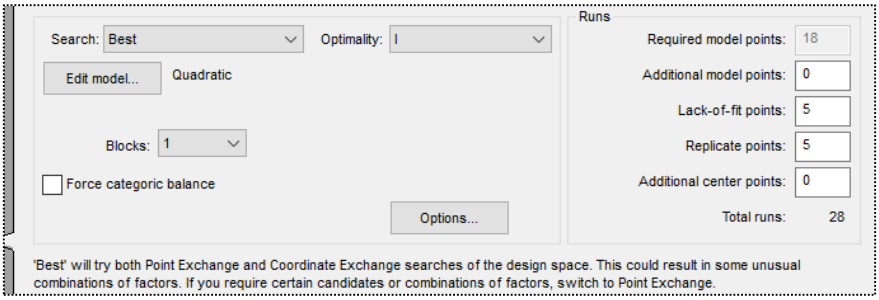
Click the yellow **Response Surface** folder tab, select **Optimal (custom)** as your design. For **Numeric Factors** choose **3** from the droplist. For **Categoric Factors** choose **1**. Key in (and tab over) **Name**, **Units**, **L[1]** (lower limit) and **L[2]** (upper limit) for numeric factors A, B, and C as shown below. For categoric factor D, key in (and tab over) **Name, Units, Levels (3),** and **L[1], L[2], L[3]**.




*Entering numeric and categoric factor levels for optimal design*

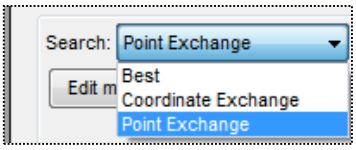
In the above screen shot, the **Type** column automatically selects via default either "Nominal" or "Continuous." "Nominal" applies to categoric factor D because its treatments are discrete names (S1, S2, S3). When entering ordered treatments (1, 2, 3, for example), it is best to set these up with "Ordinal" contrasts – available as a droplist option – because subsequent model construction and ANOVA layout are then properly applied.

Press **Continue.** Notice the "Search" droplist defaults to "Best" and "Optimality" defaults to "I."



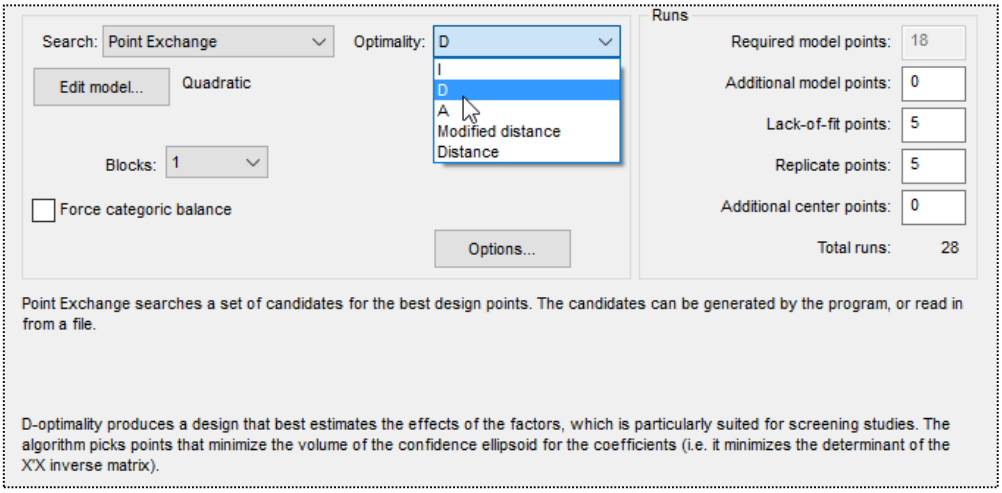

*Default selections for optimal design*

To create more convenient levels in the numerical factors, change the **Search** to **Point exchange**.



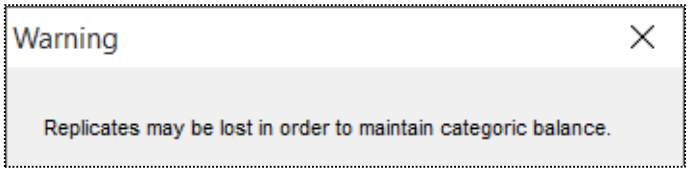

*Changing search to point exchange*

Change the **Optimality** to **D**, which may be the better choice for designs with categorical factors. For more statistical details on Design-Expert's optimal model selection screen press the screen tips icon (the light bulb) or select Tips, Screen Tips off the main menu.



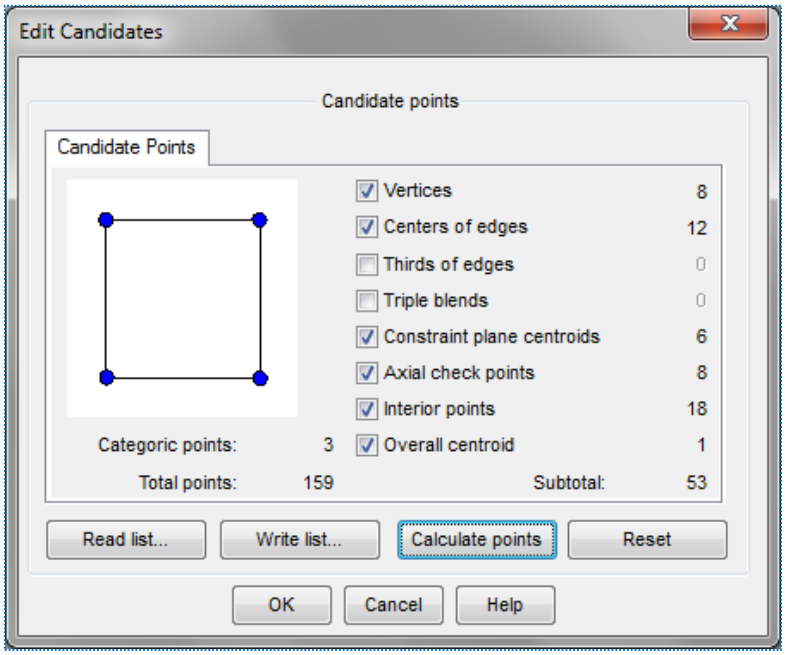

*Changing to D optimality*

At this stage the runs total to 28, which is not divisible by 3 – desirable for splitting evenly among your three suppliers. To compensate for this, bulk up your design to 36 runs (12 to each supplier) by increasing **Lack-of-fit points** to **9.** Tab to **Replicate points** and again enter **9**. Set a check-mark at **Force categoric balance**. Not doing this could result in a design with more of one supplier than another, which would doubtless generate questions from the purchasing department because it would seem unfair. A warning alerts you that forcing categoric balance might require adjustments.




*Forcing categoric balance might require adjustments*

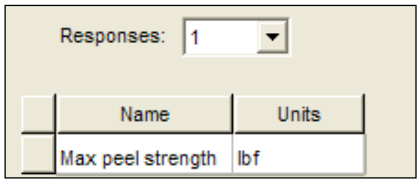
Press **OK** to continue. Now press **Edit candidate points** so you can see which combinations Design-Expert starts with, from which only an optimal subset will be chosen. A pop-up screen appears. Press **Calculate points** to see if the point selection produces more than the minimum needed to fit the selected model.




*Optimal specifications modified*

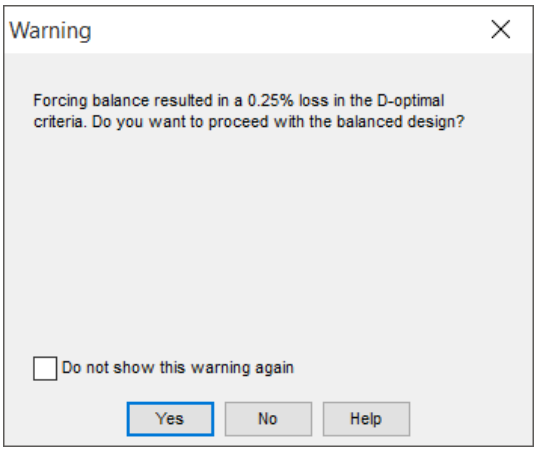
The program now identifies 53 candidate points for each of the 3 suppliers for a total of 159, from which it will select optimally 18 points needed at a minimum for the quadratic model. Design-Expert will add 9 more runs with unique factor combinations to test lack of fit. Finally, 9 of the 27 points already identified will be replicated.

Press **OK**, then press **Continue**. Enter response **Name** and **Units** as shown below.




*Response names and units – name field expanded horizontally*

Press **Finish**. A warning may alert you Design-Expert is preserving a balanced design for you, which causes a slight loss in optimality.

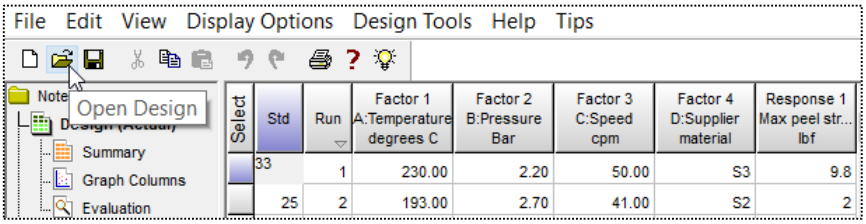



*Forced-balancing warning about a slight loss in optimality*

Your loss percentage may differ from that shown below, but in any case it will be negligible. If the warning appears, choose **Yes** to bring up the design template.

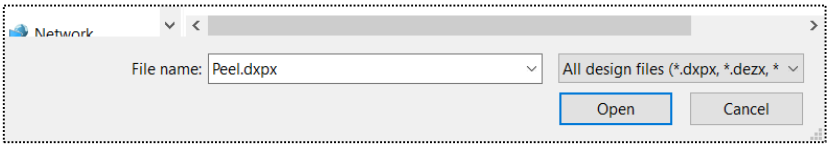
# Analyze the Response

Your layout may match what's shown in the original data table, but there are actually many possible and equally good point selections. To avoid the possibility of a mismatch, but more importantly, to save you time in entering responses, click the open design icon (the one that looks like a folder that's shown below) or select **File, Open Design.**



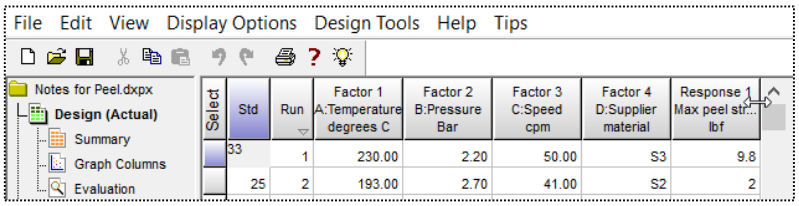

*Opening a design*

Find the file named **Peel.dxp** and open it. If you are asked to save changes to "MyDesign.dxp", choose no.



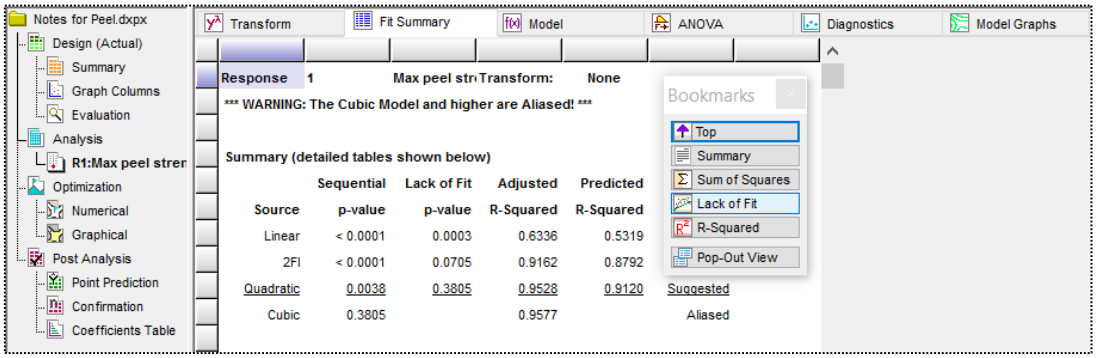

*Opening the file previously created for this case*

Notice that the header for the Response 1 column is truncated (Max peel str…). To expand the width of the column automatically, just mouse over the right border until you get the double sided arrow (shown below) and double-click. Now, the whole column header (Max peel strength) can be seen.



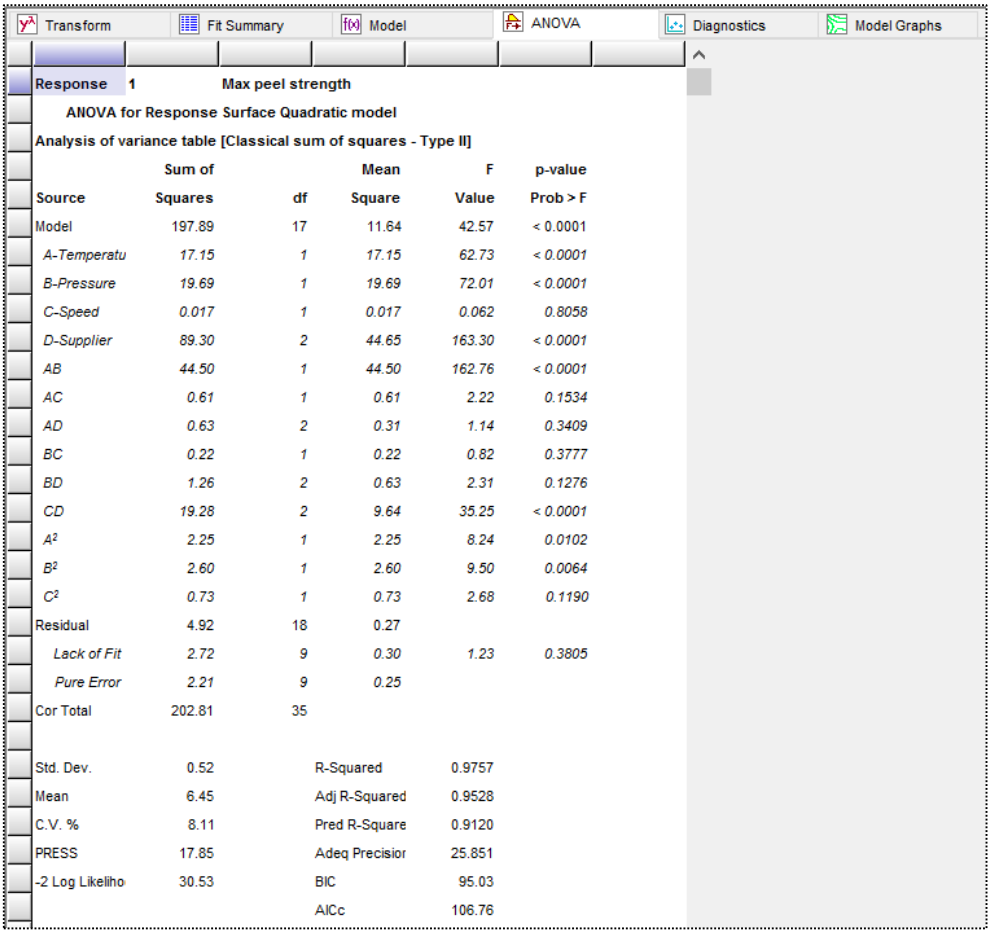

*Automatically adjusting column width using the double arrow*

Under the **Analysis** branch, click **Max peel strength**. Then press ***Fit Summary*** atop the screen.




*Fit summary report*

Notice that design-for model, quadratic, is suggested. On the **Bookmarks** palette, click **Lack of Fit**. This looks good (insignificant at p>0.1), so press ahead to the **Model** button, then ***ANOVA*** with this chosen model.



Transform | Fit Summary | Model | ANOVA | Diagnostics | Model Graphs

**Response 1 Max peel strength**

**ANOVA for Response Surface Quadratic model**

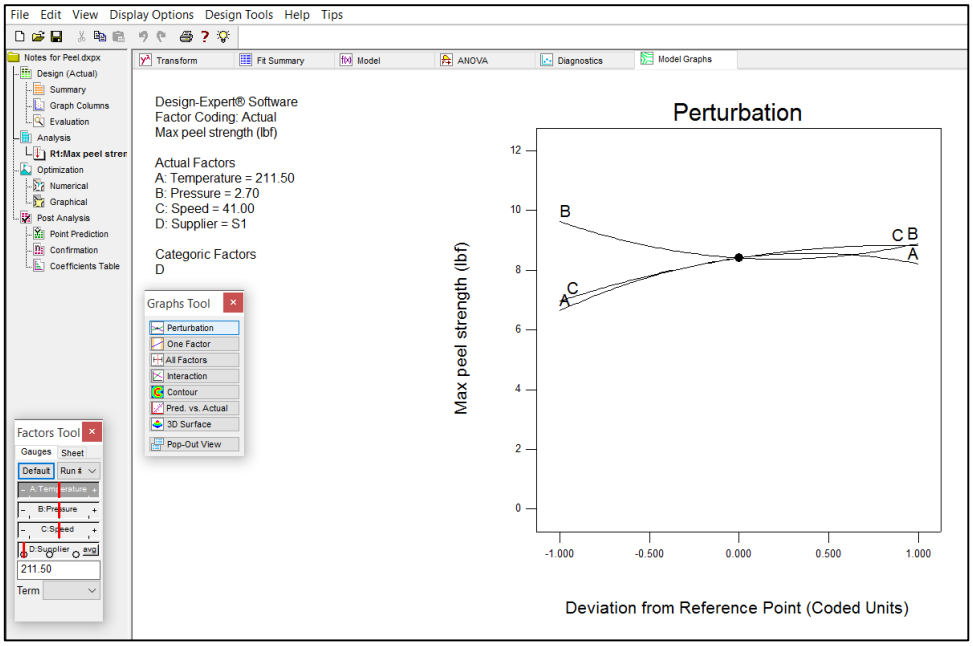
**Analysis of variance table [Classical sum of squares - Type II]**

| Source | Sum of Squares | df | Mean Square | F Value | p-value Prob > F |
|---|---|---|---|---|---|
| Model | 197.89 | 17 | 11.64 | 42.57 | < 0.0001 |
| *A-Temperatu* | *17.15* | *1* | *17.15* | *62.73* | *< 0.0001* |
| *B-Pressure* | *19.69* | *1* | *19.69* | *72.01* | *< 0.0001* |
| *C-Speed* | *0.017* | *1* | *0.017* | *0.062* | *0.8058* |
| *D-Supplier* | *89.30* | *2* | *44.65* | *163.30* | *< 0.0001* |
| *AB* | *44.50* | *1* | *44.50* | *162.76* | *< 0.0001* |
| *AC* | *0.61* | *1* | *0.61* | *2.22* | *0.1534* |
| *AD* | *0.63* | *2* | *0.31* | *1.14* | *0.3409* |
| *BC* | *0.22* | *1* | *0.22* | *0.82* | *0.3777* |
| *BD* | *1.26* | *2* | *0.63* | *2.31* | *0.1276* |
| *CD* | *19.28* | *2* | *9.64* | *35.25* | *< 0.0001* |
| *$A^2$* | *2.25* | *1* | *2.25* | *8.24* | *0.0102* |
| *$B^2$* | *2.60* | *1* | *2.60* | *9.50* | *0.0064* |
| *$C^2$* | *0.73* | *1* | *0.73* | *2.68* | *0.1190* |
| Residual | 4.92 | 18 | 0.27 | | |
| *Lack of Fit* | *2.72* | *9* | *0.30* | *1.23* | *0.3805* |
| *Pure Error* | *2.21* | *9* | *0.25* | | |
| Cor Total | 202.81 | 35 | | | |

| | | | |
|---|---|---|---|
| Std. Dev. | 0.52 | R-Squared | 0.9757 |
| Mean | 6.45 | Adj R-Squared | 0.9528 |
| C.V. % | 8.11 | Pred R-Square | 0.9120 |
| PRESS | 17.85 | Adeq Precisior | 25.851 |
| -2 Log Likeliho | 30.53 | BIC | 95.03 |
| | | AICc | 106.76 |

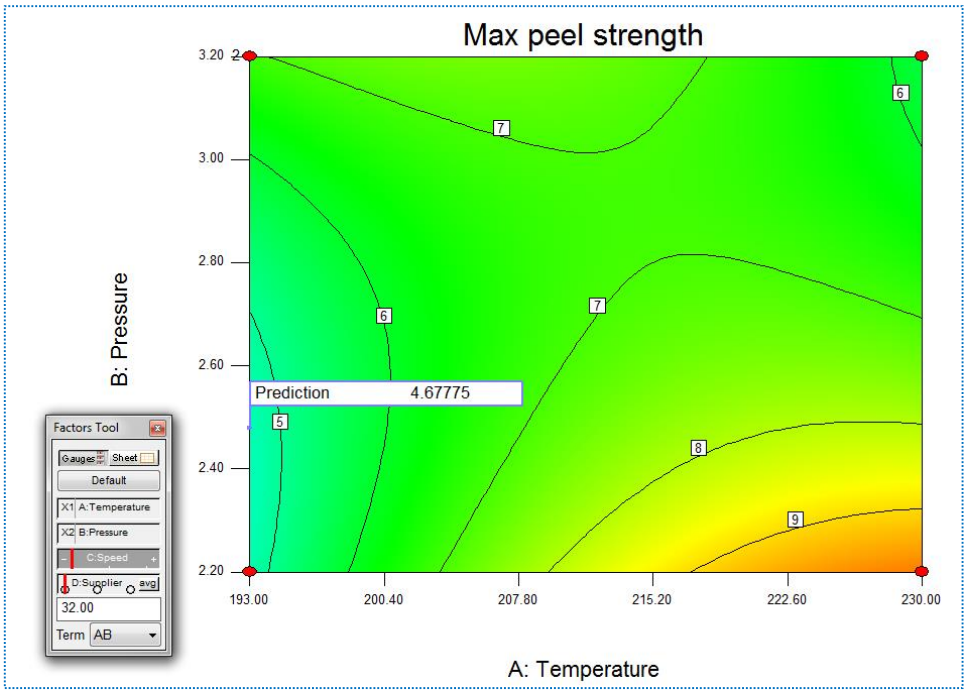
*ANOVA for quadratic model*

The statistics are excellent. Some terms come out insignificant, but carry them along by pressing ***Diagnostics*** (look these over – they shouldn't alarm you). Now view ***Model Graphs***. This is where it gets intriguing because the suppliers and purchasing agent are anxious to see the findings! Select **Perturbation** from the floating factors tool (or click this option from the view menu) to see an overview plot before generating contours and 3D surfaces.



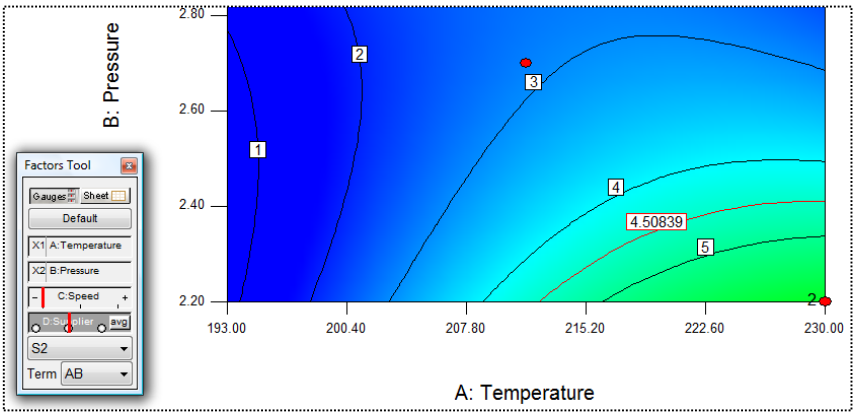

*Perturbation plot*

Remember that 4.5 lbf is the target, so supplier S1 – chosen by default – is coming out high. Lowering factor C will help bring the response down towards the target value. Return to **View**, **Contour** (or click this plot option on the Graphs Tool). On the **Factors Tool,** click **C:Speed** and slide it left. At the lowest response levels – the defaulting blue-green graduated color shading – see what's predicted via a right-click and **Add flag.**



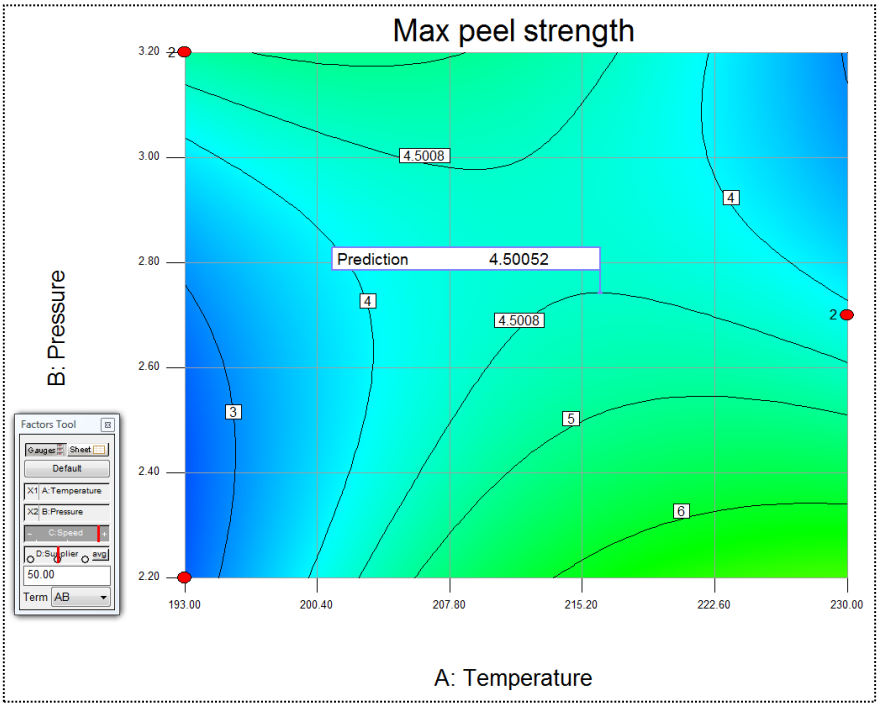

*Supplier 1 is not able to produce the target response, even at lowest speed (factor C)*

On the **Factors Tool** at **D:Supplier,** click the middle dot to see S2 results. Then right-click the graph, **Add Contour,** and drag it near the required 4.5.



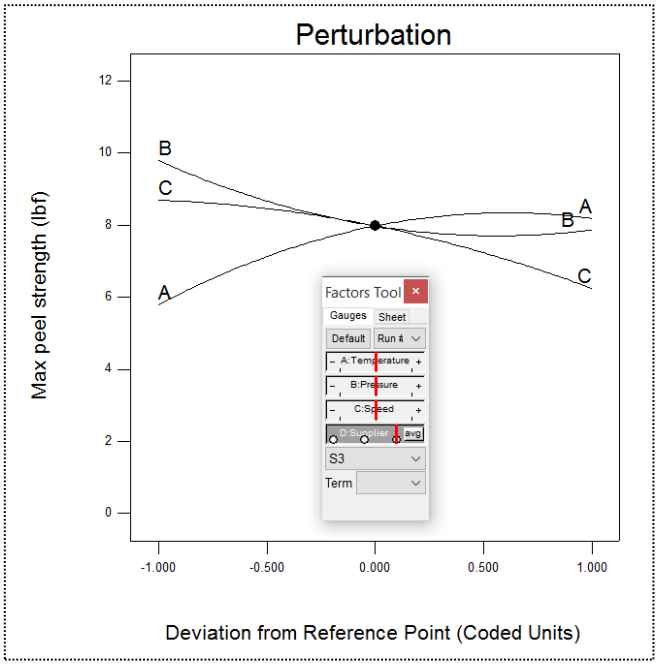

*Supplier 2 contour plot*

On the **Factors Tool,** click **C:Speed** and slide it right. Then right-click and **Add flag** near the middle that achieves the response specification. This second supplier (S2) looks much better. With their material, the process need not be pushed too hard in one factor direction or another to meet the critical specification of maximum peel strength on their packaging material.



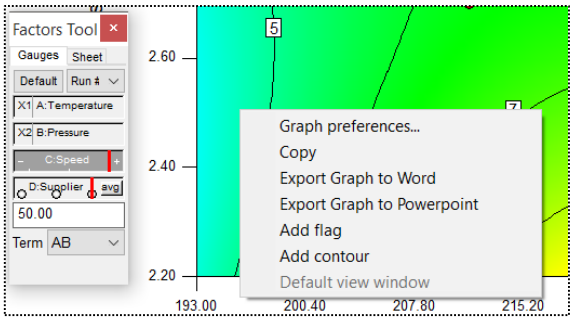

*Supplier 2 is looking good!*

On the **Factors Tool** at **D:Supplier,** click the right dot to see S3 results – it doesn't look good – the peel strength goes too high again. ☹ Select from the Graphs Tool the **Perturbation** plot to get an overview of what's occurring with this supplier. Click **Default** on the **Factors Tool** to center **C:Speed.** Then click the right dot at **D:Supplier** to reset it for S3.



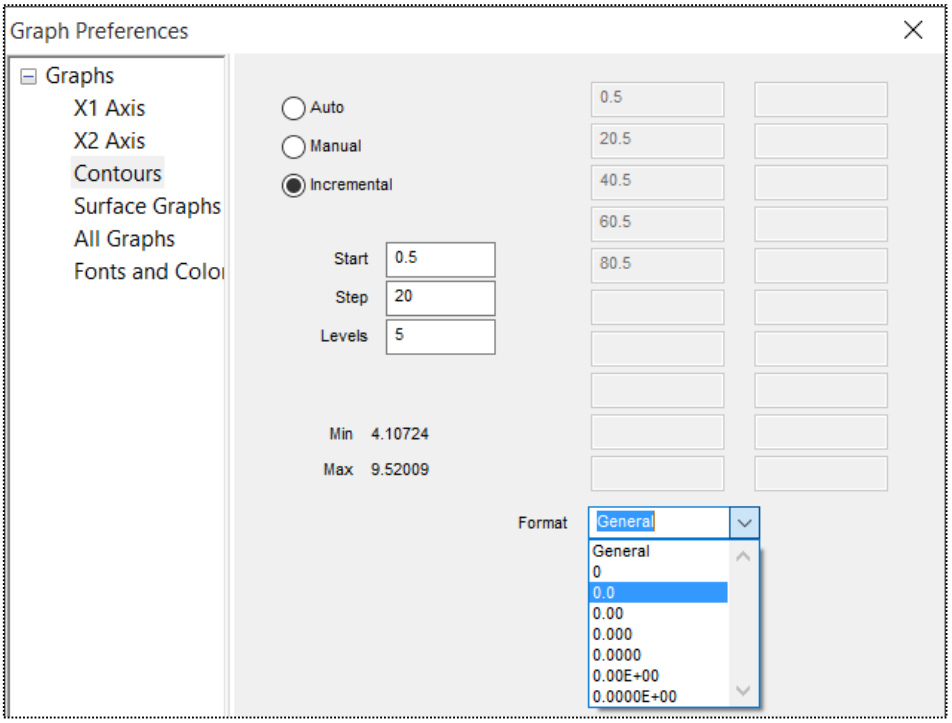

*Perturbation plot for supplier S3*

Notice the high factor C level might allow the 4.5 target to be achieved at certain A and B settings. Go back to **View**, **Contour,** and on the **Factors Tool** click **C:Speed** and slide it right. Then right-click the graph and select **Graph preferences**.



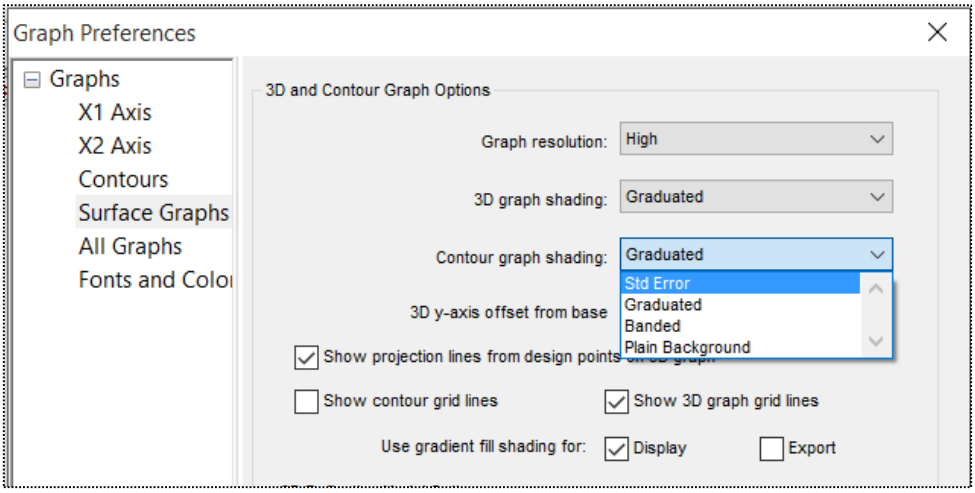

*Third supplier chosen for contour plot with Graph preferences selected*

On the **Contours** tab for graph preferences, select **Incremental** option. Enter a **Start** of **0.5**, **Step** of **0.5**, and set **Levels** at **20** (the maximum). Then change **Format** to **0.0**.




*Changing contour levels*

On the **Surface Graphs** tab, select for **Contour graph shading** the **Std Error** shading option (better for black and white printing).

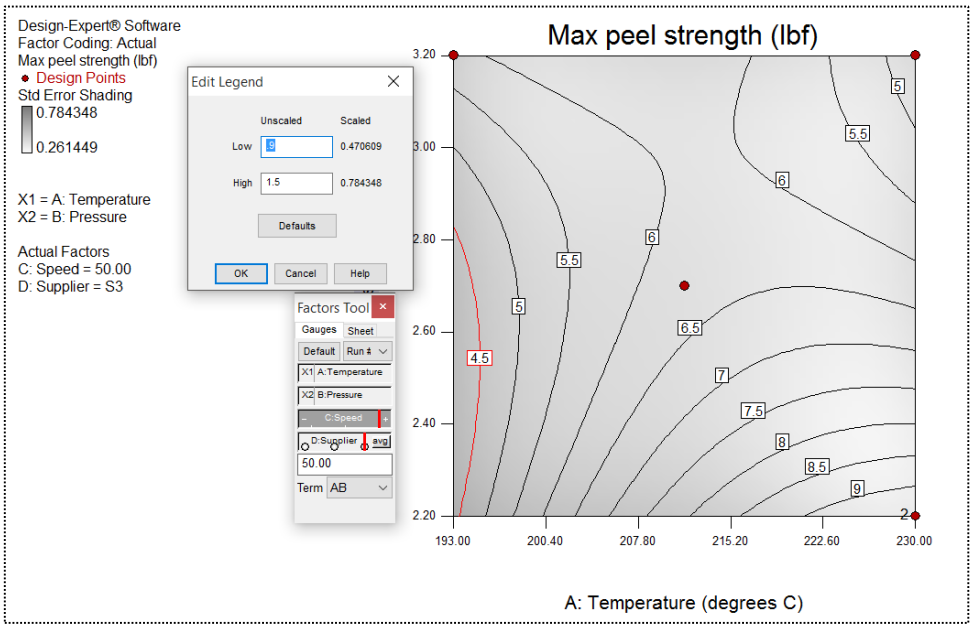



*Standard error shading selected for contour graph*

Press **OK.** Click the 4.5 contour line and note where the specification is met by this supplier with the proper process setup. It becomes a red line for easy viewing and tracking.

The contrast on the graph is not sufficient to see what's going on, so click on the shading bar in the legend. In the Edit Legend dialogue box (see picture below) change the high value to 0.9. This will cause the grey shading to get darker at a lower level, creating more contrast. This trick can be used with a color contour

plot, too. Adjusting the high and low levels on the shading bar can create better looking plots with more colors.




*The target of 4.5 highlighted as a contour for supplier 3. Right-click the shading bar to edit the legend and change the high to 0.9*

The red dots represent conditions where actual experimental runs were performed and, in one case, replicated (as indicated by the "2"). Unfortunately, no points are seen near the desired contour. Thus this region is shaded darker by Design-Expert to indicate a higher standard error for predictions.

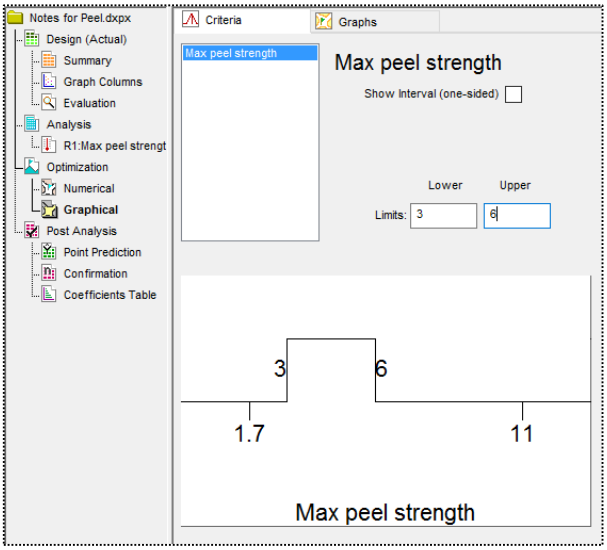
# Graphical Optimization

Recall the questions asked at the outset of this tutorial about the new packaging machine developed by the engineers at P&G. We now know the answer to the first one – yes, the process can be adjusted to hit the maximum peel strength target of 4.5 lbf for one or more suppliers.

The purchasing agent intended to break down the supply in this manner: 50% to S1, 25% to S2, and 25% to S3. This led to the second question: Can you find a process setup that works for all suppliers? The contour plots for S1 make it clear that this supplier, despite being favored by the purchasing agent, will not achieve targeted peel strength within current factor levels. That foils the next most acceptable alternative of a two-supplier option with S1 being one of them. (For some reason, hopefully based on cost, the purchasing agent really likes the first supplier!)

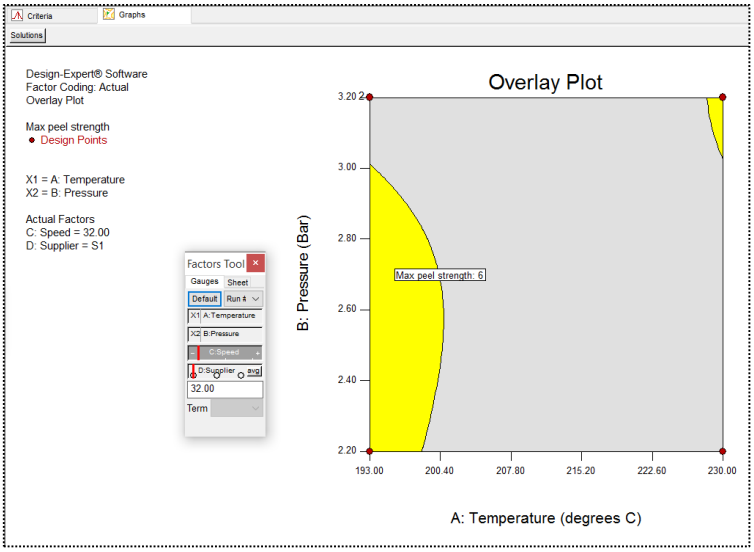
All that's left for the engineers is the following fallback position: If it is not possible to achieve a common setup for even two of the three suppliers, perhaps the upper limit of 6 could be raised on the maximum peel strength specification. (Assume your customers are willing to use scissors or, better yet, package designers can notch the plastic for easier opening.) Will this open a window of operability for multiple suppliers using one set of process conditions?

It's time to explore the operating windows of each of the three suppliers for making packaging that exceeds 3 lbf (to prevent leaking) and not exceed 6 lbf (making the package too difficult to open). Under the **Optimization** branch, click the **Graphical** node. Enter the **Limits** at **Lower 3** and **Upper 6**.



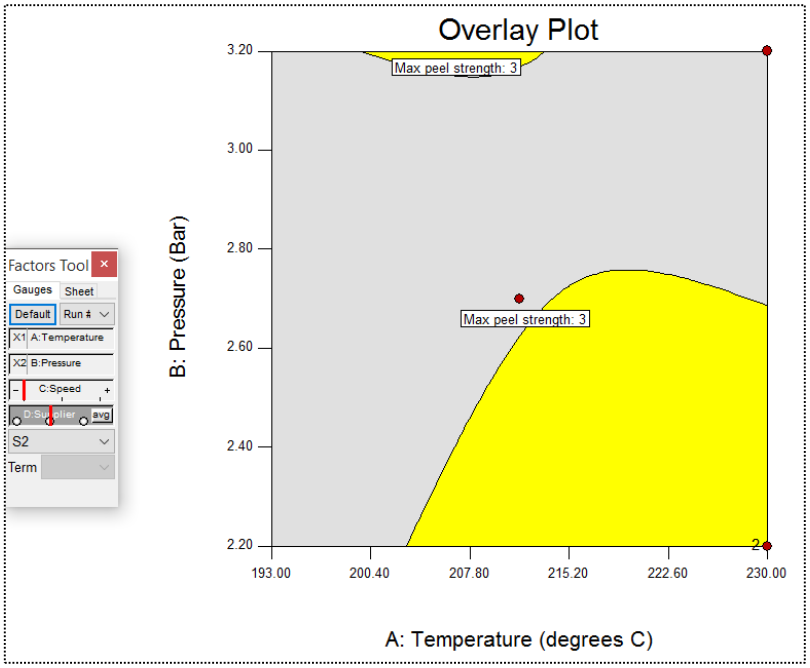

*Criteria for graphical optimization*

Click the ***Graphs*** button. You'll see there's no operating window at the default settings. On **Factors Tool** you must therefore click the bar for **C:Speed** and slide it left, thus opening a sweet spot for favored supplier (S1).



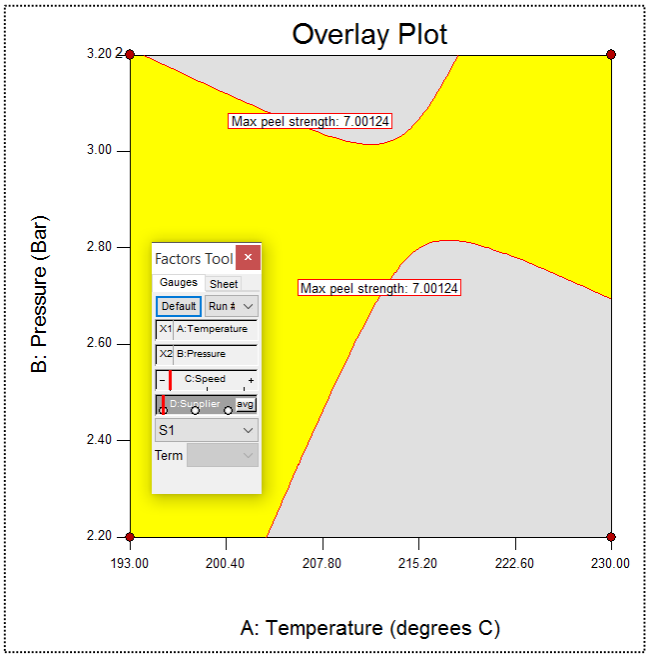

*Sweet spot for supplier 1 at low speed (C)*

Next click **D:Supplier** dots for alternatives S2 (middle) and S3 (right). Notice only S2 works at this level of C, but it offers no window of opportunity co-existing with S1.




*Window for supplier 2 at same level of factor C*

Now the only remaining alternative is to click the **D:Supplier** dot for S1 (left dot), then click and drag the border for Max peel strength to the right, causing it to increase above 6. Go ahead and push it all the way up to near 7. (Better have your scissors handy for opening the package!)




*Dragging the border up to 7 for max peel strength*

That's far enough into this case study for tutorial purposes. If you like, click through the other suppliers to see if there's a common condition where even the newly raised packaging specification can be achieved. The answer may surprise you.